**St Stephen's School – Carramar Campus**
**Year 11 Mathematics Specialist Test 2 part A**
**Total Marks: 29 Time Allowed: 30 mins Resource Free**

**Name:** _____ **Mark:**

**Teacher:**_____

**Parent Signature:**

**INSTRUCTIONS**
Permitted equipment:

- No calculators are permitted
- No notes may be used
- Stationery and drawing equipment
- Refer to the attached formula sheet

**Question 1. [4+2=6 marks]**
Forces $F_1$ and $F_2$ act on a body. $F_1 = (5\mathbf{i} - 7\mathbf{j})$N and $F_2 = (-2\mathbf{i} + 3\mathbf{j})$N

a. Find the ***magnitude*** of the resultant force acting on the body.

$F_1 + F_2 = 5i - 7j + -2i + 3j$

$= 3i - 4j$ √√

$|F_1 + F_2| = |3i - 4j|$

$= \sqrt{3^2 + 4^2}$ √

$= 5$ N √

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b. What force $F_3$ could be applied to the body to return it to equilibrium? (Give your answer in component form.)

$F_3 = -(F_1 + F_2)$

$= -(3i - 4j)$

$= -3i + 4j$ √√

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**Question 2.** **[1+1+2+2=6 marks]**

Evaluate exactly:

a. $\log_2 32$ $= 5$ ✓ 5

b. $\log_{10} 0.01$ $= -2$ ✓ 5

c. $\log_b \sqrt{b}$ $= \log_b b^{\frac{1}{2}}$ ✓ $= \frac{1}{2}$ or $0.5$ ✓ 5

d. $\log_2 7 - \log_2 14$ $= \log_2 \frac{7}{14} = \log_2 \frac{1}{2} = -1$ ✓ 5

**Question 3.** **[1+1+1+1=4 marks]**

Given:

$$\mathbf{r}_A = 9\mathbf{i} + 7\mathbf{j}$$
$$\mathbf{r}_B = 12\mathbf{i} - \mathbf{j}$$
$$\mathbf{r}_C = 3\mathbf{j}$$

Find:

a. ${}_B\mathbf{r}_A = \underline{r}_B - \underline{r}_A = 3\underline{i} - 8\underline{j}$ ✓ 5

b. ${}_C\mathbf{r}_A = \underline{r}_C - \underline{r}_A = -9\underline{i} - 4\underline{j}$ ✓ 5

c. ${}_B\mathbf{r}_C = \underline{r}_B - \underline{r}_C = 12\underline{i} - 4\underline{j}$ ✓ 5

d. ${}_C\mathbf{r}_B = \underline{r}_C - \underline{r}_B = -12\underline{i} + 4\underline{j}$ ✓ 5

**Question 4.** **[3 marks]**

Prove ${}_A\mathbf{r}_B + {}_B\mathbf{r}_C + {}_C\mathbf{r}_D = {}_A\mathbf{r}_D$

LHS: $(\underline{r}_A - \underline{r}_B) + (\underline{r}_B - \underline{r}_C) + (\underline{r}_C - \underline{r}_D)$ ✓

$= \underline{r}_A + (-\underline{r}_B + \underline{r}_B) + (-\underline{r}_C + \underline{r}_C) - \underline{r}_D$

$= \underline{r}_A - \underline{r}_D$ ✓ C

$= {}_A\underline{r}_D$ = RHS QED ✓

**Question 5.** **[7 marks]**

Velocity vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ are such that

- $\mathbf{v}_A = 5\mathbf{i} + a\mathbf{j}$
- $\mathbf{v}_B = b\mathbf{i} + 3a\mathbf{j}$
- ${}_B\mathbf{v}_A = 6a\mathbf{i} + (b+2)\mathbf{j}$

Determine the values of $a$ and $b$.

$\underline{v}_{B-A} = \underline{v}_B - \underline{v}_A$

$6a\underline{i} + (b+2)\underline{j} = (b\underline{i} + 3a\underline{j}) - (5\underline{i} + a\underline{j})$ ✓

$6a\underline{i} - b\underline{i} + 5\underline{i} = 3a\underline{j} - a\underline{j} - (b+2)\underline{j}$

$(6a - b + 5)\underline{i} = (2a - b - 2)\underline{j}$ ✓

$6a - b + 5 = 0$ ①

$2a - b - 2 = 0$ ② ✓

$4a + 7 = 0$ ① − ②

$a = -7/4$ ✓✓ C

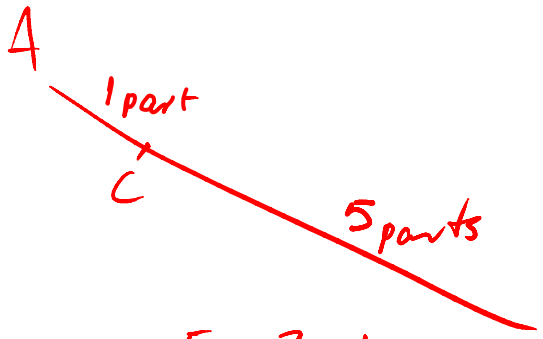
$2(-\frac{7}{4}) - b - 2 = 0$

$-\frac{7}{2} - \frac{4}{2} - b = 0$

$b = -\frac{11}{2}$ ✓✓

**Question 6. [3 marks]**

Points A and B have position vectors $\langle 5,3\rangle$ and $\langle 17,-11\rangle$ respectively. Find the exact position vector of the point C that divides AB internally in the ratio of $1:5$.




$$\underline{c} = \frac{5\times 5 + 1\times 17}{6}\underline{i} + \frac{5\times 3 + 1\times -11}{6}\underline{j} \quad \checkmark$$

$$= \frac{25+17}{6}\underline{i} + \frac{15-11}{6}\underline{j}$$

$$= \frac{42}{6}\underline{i} + \frac{4}{6}\underline{j}$$

$$= 7\underline{i} + \frac{2}{3}\underline{j} \quad \checkmark\checkmark \quad 5$$

# Formula Sheet

This formula sheet may be used for both parts of test 2.

## Component vectors

If $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$

Then the magnitude of $\mathbf{u}$ is given by $|\mathbf{u}| = \sqrt{a^2 + b^2}$

and the direction of $\mathbf{u}$ is given by $\tan\theta = \frac{b}{a}$

Let $\mathbf{u}$ be a vector of magnitude $r$ and direction $\theta$ (measured anti-clockwise from the positive $x$-axis).

Then $\mathbf{u}$ can be written in component form as $\mathbf{u} = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j}$

## Relative vectors

$$_{\mathrm{P}}\mathbf{r}_{\mathrm{Q}} = \mathbf{r}_{\mathrm{P}} - \mathbf{r}_{\mathrm{Q}}$$

## Index laws

$$a^n \times a^m = a^{n+m}$$
$$a^n \div a^m = a^{n-m}$$
$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}$$
$$(a^n)^m = a^{n\times m}$$
$$(ab)^n = a^n \times b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
$$a^1 = a$$
$$a^0 = 1$$

## Logarithms and Indices

$$\text{If } a^x = b \text{ then } \log_a b = x$$

## Log laws

$$\log_a(bc) = \log_a b + \log_a c$$
$$\log_a \frac{b}{c} = \log_a b - \log_a c$$
$$\log_a(b^n) = n\log_a b$$
$$\log_a a = 1$$
$$\log_a 1 = 0$$

## Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$